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A SYMPOSIUM OF DISCUSSION ON THE NATIONAL COMMITTEE REPORT ON JUNIOR HIGH SCHOOL MATHEMATICS.

For the convenience of the reader, parts IV, V, and VI of the National Committee Report on Junior High School Mathematics are reprinted here. The complete report, *Secondary School Circular No. 6*, July, 1920, may be secured from the U. S. Bureau of Education. Following the reprint are five discussions of the report.

IV. General Considerations Concerning the Mathematics of the Junior High School.

1. The primary purpose of the teaching of mathematics should be to develop those powers of understanding and analyzing the interdependence of quantities and spatial magnitudes which are necessary to a better appreciation of the progress of civilization and a better understanding of life and of the universe about us, and to develop those habits of thinking which will make these powers effective in the life of the individual.

2. All topics, processes, and drill entering into the course must be justified with reference to the development of the powers of understanding and analyzing relations of quantity and space, and to the development of those habits of thinking, that have just been referred to.

3. At the end of the sixth school year the pupil should be able to perform the fundamental operations with integers and with common and decimal fractions with accuracy and a fair degree of speed.

4. In view of the fact that pupils may be expected to remain in school until the end of the junior high school period, instead of leaving, in large numbers at the end of the eighth school year, as at present, the mathematics if the three years of the junior high school should be regarded as a unit.

5. The course should include arithmetic, intuitive geometry,

algebra, numerical trigonometry, and an introduction to demonstrative geometry.

6. The dependence of one quantity upon another, which involves the idea of function, should be emphasized throughout.*

7. Advantage should be taken of the opportunities afforded in the solution of problems for the exercise of judgment and self-reliance, and for the development of accuracy and a fair degree of speed in computation.

8. Emphasis should be placed on the use of common sense in computing from approximate data; as, for example, to fix the idea that the area of a circle can not be found accurately to the thousandths of a square inch, if the measurement supplies only the number of whole inches in the diameter. Pupils should be encouraged to use interest tables, tables of squares, square roots, etc., in computation.

9. The systematic development by observation, measurement, and construction of the elementary properties and relations of geometric figures should lead gradually to methods of abstract and formal proof.

10. The applications of arithmetic to business should be continued late enough in the course to bring to their study the pupil's greatest maturity, experience and mathematical knowledge, and to insure real significance of this study in the business and industrial life that many of the pupils will enter upon at the close of the junior high school period.

11. The teachers needed for the mathematics classes of the junior high school can probably be secured most advantageously by selection from the ranks of successful teachers of the upper grades of the elementary school. These selected teachers should undergo a course of training in the subject matter and methods of high-school mathematics before entering upon junior high-school teaching. In the future it will be highly desirable to require special training for teachers preparing to teach in the junior high school. It should be clear that the work will not attain the standard desired unless the teacher knows the subject of high-school mathematics thoroughly and sees the need for vitalizing the work and relating it to the interests and experiences of the pupils.

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* A special report on the rôle of the function concept in secondary school mathematics will be published in the near future.

12. Arithmetic should not be completed before the pupil has acquired the power of using algebra as an aid.

13. Intuitive geometry should be introduced before algebra is taken up as a general topic, although algebraic processes should be introduced as needed in the work in mensuration.

14. Numeral trigonometry should be based upon algebra and intuitive geometry, rather than upon demonstrative geometry.

15. It is desirable that each pupil should, upon leaving school, know the significance of a demonstration in geometry. Such demonstrative geometry as is offered may properly come rather late in the course, since it requires more intellectual maturity than is needed in the other work.

V. MATERIAL OF THE COURSE IN MATHEMATICS FOR THE JUNIOR HIGH SCHOOL.

In the course in mathematics for the junior high school outlined in the following pages no attempt is made to prescribe the order of presentation of the topics included. Much experimentation must precede any formulation of content and order that can make any claims to validity and authority.

The following recommendations as to content and arrangement of the course of study are intended to form the basis of study, discussion, and classroom experiment. With the cooperation of the supervisors of junior high schools, and their mathematics teachers, serious and constructive criticism, based on actual classroom experience with junior high-school pupils, may be gathered by the National Committee on Mathematical Requirements, and made available to all interested in the development of the junior high school. The outcome of this cooperation, with the national committee serving as a clearing house of ideas and material, should be highly significant for the teaching of mathematics in this type of school.

It is felt that this experimentation will not produce the best results if this committee should attempt, at the present time, to specify, except in a very general way, the topics to be considered in the several school years (see Sec. VI of this report).

Stated by topics rather than years, the mathematics of the junior high school may properly be expected to include the following:

A. Arithmetic:

- (*a*) The fundamental operations of arithmetic.
- (*b*) Tables of weights and measures in general practical use.
- (*c*) Emphasis on simple fractions: ½, ⅓, ⅔, ¼, ¾, ⅕, ⅛. Fractions other than these to have less attention.
- (*d*) Facility and accuracy in the four fundamental operations; time tests, taking care to avoid subordinating the teaching to the tests, or to use the tests as measures of the teacher's efficiency.
- (*e*) Very simple short cuts in multiplication and division (such as replacing multiplication by 25 by multiplying by 100 and dividing by 4).
- (*f*) Percentage. Interchanging common fractions and per cents., finding any per cent. of a number, finding what per cent. one number is of another, finding a number when a certain per cent. of it is known; such applications of percentage as come within the student's experience.
- (*g*) Line, bar, and circle graphs to be used wherever they can be used to advantage; these not to be taught as a separate topic.
- (*h*) Arithmetic of the home: Household accounts, thrift, simple bookkeeping, methods of sending money, parcel post.
  Arithmetic of the community: Property and personal insurance.
  Arithmetic of civil life: Taxes.
  Arithmetic of banking: Savings accounts, checking accounts, foreign money.
  Arithmetic of investment: Real estate, elementary notions of stocks and bonds, postal savings.
- (*i*) Statistics. Fundamental concepts; statistical tables and graphs; pictograms; simple frequency distributions; measures of central tendency.

B. *Intuitive Geometry:*

- (*a*) The direct measurement of distances and angles by means of a linear scale and protractor. The approximate character of measurement. The degree of precision as expressed by the number of significant figures.
- (*b*) Indirect measurement by means of drawings to scale. Uses of square ruled paper.
- (*c*) Areas of the square, rectangle, parallelogram, triangle, and trapezoid; circumference and area of a circle; surfaces and volumes of solids of corresponding importance.
- (*d*) Practice in numerical computation with due regard to the relation between the precision of the data and the significant figures in the result.
- (*e*) Simple geometric constructions with ruler and compasses, T-square and triangle, such as perpendicular bisector, angle bisector, parallel lines, etc.
- (*f*) Familiarity with such forms as the equilateral triangle, the 30°–60° right triangle, and the isosceles right triangle; symmetry, axial

and central; a knowledge of such facts as those concerning the angle sum for the triangle and the Pythagorean relation; simple cases of geometric loci in the plane and in space.

(*g*) Geometry of appreciation: Geometrical forms in nature, architecture, manufacture, and industry.

The work in intuitive geometry should make the pupil familiar with the elementary ideas concerning geometric forms in the plane and in space with respect to shape, size, and position. It should, moreover, be carefully planned so as to bring out geometric relations and logical connections. Before the end of this intuitive work the pupil should have very definitely begun to make inferences and draw valid conclusions from the relations discovered. In other words, this informal work in geometry should be so organized as to make it a gradual approach to, and provide a foundation for, the subsequent work in formal demonstrative geometry.

C. Algebra:

1. The formula—its construction, meaning, and use—
   - (*a*) As a concise language.
   - (*b*) As a shorthand rule for computation.
   - (*c*) As a general solution.
   - (*d*) As an expression of the dependence of one variable on other variables.

The work done with the formula will include translation from English into algebraic language, and vice versa.

2. Graphs and graphic representations in general—their construction and interpretation in—
   - (*a*) Representing facts (statistical, etc.)
   - (*b*) Representing dependence.
   - (*c*) Solving problems.
3. Positive and negative numbers—their meaning and use—
   - (*a*) As expressing both magnitude and one of two opposite directions or senses.
   - (*b*) Their graphic representation.
   - (*c*) The fundamental operations applied to them.
4. The equation—its use in solving problems—
   - (*a*) The linear and the "pure" quadratic equation in one unknown; their solutions and applications.
   - (*b*) Equations in two variables, with numerous concrete illustrations.
   - (*c*) A simple treatment of proportion. To include various simple applications of ratio and proportion in cases in which they are generally used in problems of ordinary life. In view of the usefulness of the ideas and training involved, this subject may also properly include simple cases of variation.
5. Algebraic technique:
   - (*a*) The fundamental operations.

Their connection with the rules of arithmetic should be clearly brought out and made to illuminate arithmetical processes. Drill in these operations should be limited strictly in accordance with Principles 1 and 2, mentioned in

Section IV above. In particular, nests of parentheses should be avoided, and multiplication and division should not involve much beyond monomial and binomial multipliers, divisors, and quotients.

(*b*) Factoring. The only cases that need be considered are—

  (i) Monomial factors.
  (ii) The difference of two squares.
  (iii) The square of a binomial.

(*c*) Fractions.

Here again the intimate connection with the corresponding processes of arithmetic should be made clear and should serve to illuminate such processes. The four fundamental operations with fractions should be considered only in connection with simple cases, and should be applied constantly throughout the course to gain the necessary accuracy and facility. The most difficult complex fractions taken up should contain only numerical fractions in numerator and denominator.

5. Algebraic technique—Continued.

(*d*) Exponents and radicals. The work done on exponents and radicals should be confined to the simplest material required for the treatment of formulas.

(*e*) Stress should be laid upon the need for checking solutions.

(*f*) Optional topics. Logarithms and the slide rule may be taken up with some classes. When either of these topics is included, it is recommended that discussion of the underlying theory be omitted. The subject may be taken up in connection with arithmetic or trigonometry.

D. Numerical Trigonometry:

(*a*) Definition of sine, cosine, and tangent.
(*b*) Their elementary properties as functions.
(*c*) Their use in solving problems involving right triangles.
(*d*) The use of tables of these functions (three or four places).

The introduction of the elementary notions of trigonometry into the earlier courses in mathematics has not been as general in the United States as in foreign countries. Among the reasons for the early introduction of this topic are its great practical usefulness for many citizens; the insight it gives into the nature of mathematical methods, particularly those concerned with indirect measurement; the rôle that mathematics plays in the life of the world; the fact that it is not difficult and that it offers wide opportunity for concrete and significant application; and the interest it arouses in the pupils.

E. Demonstrative Geometry.

The demonstration of a limited number of propositions, with no attempt to limit the number of fundamental assumptions, the sole purpose being to show to the pupil what ''demonstrations'' means.

Many of the geometric facts previously inferred intuitively may be used as the basis on which the demonstrative work is built. This is not intended to preclude the possibility of giving at a later time rigorous proofs of some of the facts inferred intuitionally. It should be noted that from the strictly logical point of view the attempt to reduce to a minimum the list of axioms, postulates, or assumptions is not at all necessary, and from a pedagogical point of view such an attempt is very undesirable. It is necessary, however, that those propositions which are to be used as the basis of subsequent formal proofs be explicitly listed and their logical significance recognized.

VI. SUGGESTED ARRANGEMENTS OF MATERIAL.

It is not the intention of the committee to suggest a single course of study either by years or by half years, or to recommend any artificial separation of topics; nevertheless, it is felt that a few possibilities in the arrangement of material may be helpful to teachers in deciding upon a course that is suited to their needs. The following plans for the distribution of time are therefore suggested, but no one of them is recommended as superior to the others, and only the large divisions of material are mentioned.

Plan A.

First year: Applications of arithmetic, particularly in such lines as relate to the home, to industry, to thrift, and to the various school subjects; intuitive geometry.

Second year: Algebra; applied arithmetic, particularly in such lines as relate to the commercial, industrial, and social needs of our country.

Third year: Algebra, trigonometry, demonstrative geometry. (By this plan the demonstrative geometry is introduced in the third year and arithmetic is practically completed in the second year.)

Plan B.

First year: Applied arithmetic (as in Plan A); intuitive geometry.

Second year: Algebra, intuitive geometry, trigonometry.

Third year: Applied arithmetic, algebra, trigonometry, demonstrative geometry.

By this plan trigonometry is taken up in two years, and the arithmetic is transferred from the second year to the third year.

Plan C.

First year: Applied arithmetic (as in Plan A); intuitive geometry; algebra.

Second year: Algebra; intuitive geometry.
Third year: Trigonometry; demonstrative geometry; applied arithmetic.
By this plan algebra is confined chiefly to the first two years.

Plan D.

First year: Applied arithmetic (as in Plan A); intuitive geomtery.
Second year: Intuitive geometry; algebra.
Third year: Algebra; trigonometry; applied arithmetic.
By this plan demonstrative geometry is omitted entirely.

Plan E.

First year: Intuitive geometry; simple formulas; elementary principles of statistics; arithmetic (as in Plan A).
Second year: Intuitive geometry; algebra.
Third year: Intuitive geometry; numerical trigonometry; arithmetic.
By this plan the work of the first two and one-half years may be described as general mathematics, while the last half-year would be devoted to the special civic and economic features of business practice.

DISCUSSION OF PART IV.

There was never an opportunity to work out a course in secondary mathematics as is now offered in the junior high-school movement. Assuming only the fundamental operations of arithmetic as prerequisites, with no traditions to interfere with the formation of the course, we are entirely free to select what material seems best fitted for our purposes,

As the primary purpose of the teaching of secondary mathematics the committee states "the development of the powers of understanding and analyzing the interdependence of quantities and spatial relations which are necessary to a better appreciation of the progress of civilization and a better understanding of life and the universe about us and to develop those habits of thinking which will make these powers effective in the life of the individual." Furthermore, the committee urges teachers "to take advantage of the opportunities for training judgment and self-reliance and for developing accuracy and a fair degree of speech in computation." Nothing is to be included in the course which cannot be justified on the basis of these aims.

It must be admitted that the success of a course in mathematics is directly proportional to the extent to which it succeeds in accomplishing these aims. They are the chief reasons why mathematics has held successfully a place on the curriculum. However, these porposes cannot be accomplished to the fullest extent unless the material included in the course is also justified on the basis of real needs in the life and studies of the adolescent boy or girl, and fits them to perform in later life the activities in the affairs of an active world. For, these performances not only call for the use of the indirect outcome of the study of mathematics, such as habits of thinking, and powers of analysis, generalization and concentration, but for direct use of mathematics in present-day social life and business.

Moreover, all material which can not be adapted to the mental development of the pupils, or lies outside of their experiences is of little or no value and should be rejected.

The committee suggests that the new course include further arithmetic, intuitive geometry, algebra, numerical trigonometry, and an introduction to demonstrative geometry. The following supplementary suggestions are offered.

1. The applications of arithmetic should be continued to be taught late in the three-year course.
2. Intuitive geometry is to be taken up early.
3. Algebraic processes and arithmetic should be studied in connection with mensuration.
4. There should be some numeral trigonometry.
5. Some demonstrative geometry might be introduced late in the course.

This material should prove to be far better than the traditional courses in arithmetic usually offered in the seventh and eighth grades. Recent experiments tend to show that pupils of that age can do sucessfully much of the work now taught in the senior high school, while they profit little by studying many of the applications of arithmetic because they are beyond their experience.

Although correlation is not advocated by the committee, the second, third and fourth points indicate the possibility, or even the desirability, of correlating algebra, geometry, arithmetic, and trigonometry. The plans of alternating these subjects after certain time intervals, or of carrying them parallel to each other, are usually not successful. The work ought to be so arranged that there is a motive, which the pupil can recognize for changing from one topic to another. Since the teacher, busy with classes and other school duties, is not in a position to put the material in the proper order, there is need for a course in which the topics are so arranged that they may be studied by a class without interruptions. The report does not show that the committee have considered the plan, at least not sufficiently to include it as one of the possible arrangements worked out in part VI of the report.

Such a course should accomplish the aims and purposes set up by the committee more easily than is possible with separate courses in arithmetic, algebra, and geometry. Topics could then be put in psychological order with the result that pupils would gain from the study of mathematics the broadest possibilities.

Correlation is rapidly growing not only in secondary mathematics but also in the junior colleges, and the committee should take a definite stand in this movement.

Besides the general aims set up by the committee the report mentions several specific aims. One of these is the emphasizing of the idea of function throughout the course. Functional relations occur even in some of the simplest applications of arithmetic. We find them in problems of cost, interest, percentage. Many of the principles of geometry express functional relations. These relations are expressed in tables, in symbols as formulas, or by graphs. They should be felt and understood by the pupils long before the term function is mentioned to them. It is a concept which takes on more and more meaning as the pupil's knowledge is enriched, and as he passes through the study of secondary mathematics on to courses in college algebra and analytical geometry. If the idea of function is emphasized thorughout the courses in the junior high school, this should be extremely beneficial to the more advanced courses, indeed it should form a powerful motive to induce the pupil to study advanced work.

Another specific aim mentioned in the report is "emphasis on the use of common sense in computing from approximate data." Even a careless examination of textbooks in arithmetic and trigonometry reveals how little attention is paid to this important subject. Answers given in problems in which the original data have scarcely two-figure accuracy, are computed to five and six figures by means of trigonometric tables. In computations of areas and circumferences of circles in which the radius is expressed to one or two figures we find frequently that a value of π is used with three or four decimal places. In all problems of mensuration the pupil should be led to consider whether or not the result is reasonable. Abbreviated multiplication and division should therefore be taught in the seventh grade until every pupil completely masters these processes.

The question of finding the right teachers for the junior high school is an important one. High school teachers are usually prepared to do this work as far as knowledge of subject matter is concerned. They have the broad outlook to teach the material contained in the course, but few have had sufficient experience with pupils of this age. A thorough understanding of this critical period of child life is absolutely necessary. On the other hand, most elementary school teachers lack the necessary academic training, and do not have the advanced viewpoint. The committee is right in advocating that a new teacher be trained, most advantageously selected from the upper grades of the elementary school, to teach the mathematics classes of the junior high school.

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DISCUSSION OF PART V.

The Committee on Mathematical Requirements is to be congratulated on its recent Preliminary Report on Junior High School Mathematics.

In Section V, suggested topics are listed under the headings of Arithmetic, Intuitive Geometry, Algebra, Numerical Trigonometry, and Demonstrative Geometry. These headings are used evidently for convenience, not with the idea that the material should be presented as such separate branches. Although it is expected that this work will be fused into a single course in Junior High School Mathematics, the general trend will be in the sequence given. That is, in the first part of the course Arithmetic will be stressed. Intuitive Geometry should come before Algebra, and Numerical Trigonometry before Demonstrative Geometry. But, certain topics in Arithmetic are best taught through Intuitive Geometry, as the tables of weights and measures [A. (b)-]. The equation in Algebra is best introduced through the formulas of mensuration. The indirect cases of percentage [A. (f)-] must be taught algebraically by the equation either in its regular or camouflaged form.

Among the suggestions under Arithmetic, a few may well be stressed, as

1. Limitation of fractions to simple ones.
2. Time tests.
3. Teaching of simple bookkeeping.
4. Stressing civic and social aspects of mathematics.
5. Use of graphs only to illuminate other mathematics.

It may be opportune to give a word of warning not to overstress such topics as the indirect cases of percentage, foreign money, stocks and bonds, and statistics.

The outline of Intuitive Geometry is well thought out. The success of this work is phenomenal if it is presented so that it is really intuitive, nor formal. There is no other phase of elementary mathematics that makes such an appeal to all types of pupils.

With the Intuitive Geometry as a background, the phases of elementary algebra, as outlined, can be easily presented, for algebraic expressions come to have a real meaning, impossible otherwise. If teachers of Algebra will make it their first concern to see that algebraic symbols and expressions mean something to the pupils, they will have need of less concern about their manipulation. When pupils recognize quadratic trinomials as areas of squares or rectangles, and their factoring as finding the dimensions, a real achievement has been made.

The Committee wisely recommended the limiting of factoring to three cases, but I hardly agree on the three. Personally, I prefer limiting it to two cases,

(1) The monomial factors,
(2) The quadratic trinomial.

By teaching the general cross product method for all such expressions, no separate process need be taught for the square of a binomial or for the difference of two squares. The one method may be applied to all three types.

The introduction of numerical trigonometry into the mathematic course of the junior cycle is highly desirable. It is necessary for a complete survey of the elementary field. Its practical usefulness appeals to pupils immensely. It offers an opportunity for immediate, interesting, and concrete application. At the same time it is decidedly more within the scope of the pupils' ability than the abstruse topics of Algebra formerly included in a ninth grade course. As one boy, brought up under the old regime of formal arithmetic, algebra, geometry and trigonometry, said "I never saw any sense to mathematics until I got to trig. If they had given me that earlier, I might have liked mathematics and learned it."

To be complete the junior cycle should give at least a glimpse into demonstrative geometry, only to let the pupil know what it is. Intuitive geometry fuses perfectly with other mathematics. *Demonstrative geometry cannot, for it is really logis, not mathematics.* But enough of it should be given at the close of the ninth grade so that the pupil may know what it is and whether or not he wishes to elect it for further study in the senior high school.

The report suggests that "Many of the geometric facts previously inferred intuitively may be used as the basis on which the demonstrative work is built." This is true, but care should be taken to choose theorems in which the pupil sees something to prove, not self-evident facts or assumptions.

When all seventh, eighth and ninth grades, whether they are organized in junior high schools or not, follow the outline recommended by the Committee, a great forward step will have been taken in the teaching of mathematics.

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DISCUSSION OF PART V.

An objective discussion of the tentative syllabus embodied in Section V of the report may properly begin with an expression of appreciation and general approval. Its brevity and directness will undoubtedly be welcome to the majority of teachers. There is, throughout, a gratifying effort to secure perspective by putting the essentials into bold relief, and to avoid the usual multiplicity of details found in other official syllabi.

Moreover, a commendable flexibility characterizes the recommendations of the National Committee. Thus, graphs are "to be used wherever they can be used to advantage." Applications of percentage are to be such "as come within the student's experience," and the customary minute specifications are omitted. In view of the prevailing educational uncertainty this latitude in the phraseology of the report is undoubtedly its great outstanding virtue, though only a temporary one. The National Committee will probably find it desirable to issue a series of supplementary reports dealing in detail with certain features of secondary mathematics which are under debate at present. There is as yet no definite national policy concerning intuitive geometry. Very few teachers understand why intuitive geometry should come early in the course, why it should precede algebra, and what methods are to be employed in teaching it. To the majority it is an uncharted sea. In arithmetic, such topics as "household accounts, thrift, simple bookkeeping, statistics," are receiving very different degrees of emphasis in the newer texts. The whole arithmetic situation is in a state of flux. Old landmarks are disappearing and the new horizon is not too distinct. Some of the recent demands in algebra, such as the insistence on the formula, on the function concept, and on more significant problem material, are still ignored in the average classroom, and even experienced teachers find it difficult to effect a satisfactory compromise between tradition and reform. Trigonometric material, even of the simplest type, is also *terra incognita* in most of our junior high schools.

The textbooks are approaching all these things with great moderation and caution. An impartial reader usually gets the impression that the newer ideas are introduced, if at all, as a concession, and that almost invariably they have no organic connection with the remainder of the text. In fact, they may even be omitted "at the discretion of the teacher," without in the least disturbing the sequence. Such slow and artificial revisions of the curriculum can hardly be our ultimate goal. Perhaps an individual author or publisher cannot be expected to submit a radical teaching program. It would be too costly a venture. And since textbooks so largely determine the work of the teachers, a modification of the usual routine can come only through a national program of readjustment, combined with much experimentation. Once this program has found general recognition, authors may safely begin to evolve really new texts.

It is clear, therefore, that a mere enumeration of topics will hardly suffice for classroom purposes. Specific illustrations and suitable problems must be included eventually, at least in certain critical phases of the work. The National Committee undoubtedly shares this feeling, for it has announced the impending publication of a separate report on "The Function Concept in Secondary School Mathematics." There should also be reports on arithmetic, on intuitive geometry, and on problem material in algebra.

Another matter of importance will require attention at an early date. I refer to the approximate time allotment necessary to secure a fair degree of proficiency in the average classroom. Investigations show that a great discrepancy exists in the time schedules of the new junior high schools. Some have supervised study, without homework, while some permit home assignments of a very flexible character. The number of recitations varies from three to five a week, and the length of recitations is subject to extreme fluctuations. Such divergent conditions of work must produce a corresponding dissimilarity of content and methods. Hence there must be an authoritative evaluation of topics in terms of lesson units. Thus, a minimum of ten lessons of 40 minutes each should be reserved for positive and negative numbers. At this point the interpretation of the syllabus is dependent on the psychology of learning and on scientific experimentation. The inexperienced teacher will be helped immensely by authentic time schedules. Here, again, the National Committee will make a valuable contribution by its forthcoming report on "Mathematics in Experimental Schools." In general, the curriculum outlined by the National Committee, with the exception of demonstrative geometry, may be completed satisfactorily within the three years of the junior high-school course.

Finally, it cannot be denied that the fluctuations mentioned above are often due to mutually exclusive and conflicting theories concerning the aims of secondary mathematics. There are those, a minority perhaps, who see in mathematics *merely* a unique tool for mental discipline. Accordingly, "nests of parentheses," complex fractions, and other non-essentials, are treated with relentless and time-consuming thoroughness. On the other hand, now and then we are told with equal conviction that in the junior high school the pupil is to be "tried out," to be "exposed" to mathematics, so that he may get a general idea "what it is all about." Radically different results must accompany these antagonistic views. Any experienced teacher knows that a *cumulative* subject like mathematics becomes a veritable nightmare of inefficiency, and a torture to pupils and teachers, unless the fundamental technique is made practically automatic. A lecture or two on graphs may be instructive and interesting, but that is a very different thing from securing a functioning familiarity with the subject. In the subsequent remarks it is assumed that a reasonable mastery of the essentials is one of the aims endorsed by the National Committee. (See Section IV of the report, especially statements 1, 2, 7.)

Special Considerations.

Arithmetic. In Section IV, 3, it is stated that "at the end of the sixth school year the pupil should be able to perform the fundamental operations with integers and with common and decimal fractions with accuracy and a fair degree of speed." If one may believe the current criticisms of our elementary schools, this desirable goal is rarely or never reached under existing conditions. The syllabus obviously sustains that opinion, for in Section V, A, topics (a), (b), (c), (d), (e), are devoted essentially to a review of the four fundamental operations. Concerning topics (f) and (h) one may quote almost verbatim Chapter III from the "Teaching of Arithmetic," by Professor D. E. Smith, especially the following passage:

"When we consider this list of topics (*i.e.*, the arithmetic topics previously enumerated by the author), we are struck by its simplicity and brevity. There is little here to try unduly the understanding of a learner. When we think that this is all that the world usually demands of the school, and that we are allowed eight years to impart this knowledge, we are led to ask ourselves why the world is not satisfied with the results. Is the difficulty with ourselves in that we include a lot of matter of relatively little value, but which consumes the time without any just return? Or do we fail to insist on the fundamentals while we are teaching the more advanced topics that find place in our schools?"

Topics (*g*) and (*i*) are very valuable additions to the customary routine. They furnish a natural bond between arithmetic and geometry. One question is pertinent. "Business applications" dominate the arithmetic of the junior high school, perhaps justly so. But should we not begin to insist that the science of number is not inherently confined to dollars and cents? It can talk many languages. In fact, it is "encyclopedic." As the curriculum of the junior high school expands, why should not arithmetic assume a wider range by giving due attention to science and to household and industrial arts?

Intuitive Geometry. Topics (*a*), (*b*), (*c*), (*d*) are concerned with measurement. Experience shows that pupils in the seventh grade can readily measure segments "to the nearest millimeter." After some practice they can be made to read results within half a millimeter. That seems to be the limit of accuracy possible with the instruments available. Hence the remarks of the syllabus concerning the "approximate character of measurment" and concerning "significant figures" must be interpreted accordingly. As to topic (*e*), one may question the obligatory use of the "T-square and triangle." Many schools will find it difficult to furnish such an equipment at this time. There is ample opportunity for efficient construction work without the usual accessories of mechanical drawing. Symmetry, mentioned in (*f*), should be a tool and not merely a relational fact. Much stress should be laid on topic (*g*), so frequently ignored altogether.

The syllabus emphasizes the study and appreciation of geometric forms, measurement and computation, constructions, and geometric relations. It does not offer suggestions as to arrangement or sequence. It might be well to inform inexperienced teachers that intuitive geometry is not primarily a textbook subject, that it requires constant contact with concrete material, and that the method of procedure is all-important.

Algebra. Perhaps the material outlined in the tentative syllabus may be summarized as follows: the equation, the graph, the formula, and the technique which is necessary to make algebra effective for purposes of computation and problem solving. Positive and negative numbers may be included under the heading of technique. This moderate outline is in harmony with modern demands. A few remarks concerning each of these main divisions may be appropriate.

1. *The equation.* Apparently the "complete" quadratic equation is to be omitted. That means the elimination of the "quadratic formula" from the elementary course. The wisdom of this suggestion depends entirely on the question whether one can find at least a few good, non-technical, applied problems that can be solved only by means of "quadratics." Equations in two variables are to be included, "with numerous concrete problems." It must be confessed that the problem material usually following these equations is not of very convincing importance. Proportion should be treated much more liberally, in view of its constant use in science and in the problems of ordinary life. This seems to be the principal opportunity to make "fractional" equations worth while. Moreover, the function concept in its simplest form finds a ready symbolic expression in the study of proportionality.

2. *The graph.* The representation of statistical data is ordinarily of a merely informational or preparatory character. The truly mathematical use of graphs is concerned with the picturing of functional relations and with the solution of problems. Statistical (non-mathematical) graphs have finally invaded the elementary books, but functional graphs thus far seem to exist chiefly for ornamental purposes.

3. *The formula.* There are those who look upon this topic as the one great justification of algebra in an elementary

course. In the shop, and in the literature intended for the general reader, the formula is said to be the only form of algebraic training found universally necessary. However that may be, for school room purposes this contention is subject to two very serious difficulties. The transformation of even ordinary formulæ often requires a respectable amount of technique not to be acquired in a moment. And the manipulation of formulæ which are meaningless to the pupil is as objectionable a bit of formalism as was the juggling of the one-sided technique of former days. The remedy must eventually be found by furnishing data from which the pupil can *derive* the underlying relations. These relations, expressed in graphic or symbolic form, can then furnish a suitable point of departure for a subsequent extension to discoveries made by others. This is training in functional thinking, so persistently demanded by the world's leading mathematicians. A good beginning in this direction has been made. (See Nunn, The Teaching, of Algebra, Chapter III. Also, Nunn, Exercises in Algebra, Part I.)

4. *Algebraic technique.* The reduction of factoring to three "cases" will be welcome to those who have long regarded with distress the really alarming amount of time that was generally found necessary for this barren field. The omission of the general trinomial is a logical sequel to the disappearance of complete quadratics. Naturally, the usual work in fractions will then be shortened very considerably. In fact, the chief use of algebraic fractions will probably center around the transformation of formulæ. The (optional) introduction of logarithms is perhaps the only means of ever making exponents seem significant to a pupil.

As was stated previously, unless the formula is to become the backbone of elementary algebra, a more promising collection of problems must be forthcoming. The National Committee may find the creation of a vitalized algebra course suitable for the average pupil in the junior high school, a domain of investigation of which only the general outlines have been discerned.

Numerical Trigonometry. This enrichment of the curriculum will occasion very little difficulty. Elementary trigonometric notions may be introduced at any time, even as early as the seventh grade. The chief precautions should be to "make haste slowly." One of the principal contributions of even a rudimentary treatment of trigonometry will be the immense increase in the range of significant problems.

Demonstrative Geometry. There are a few good American schools, usually private or endowed, that succeed in doing some excellent work in demonstrative geometry in the ninth school year. The majority of our junior high schools will probably find it impossible, for years, to reach the art of geometric demonstration. It is partly a question of mathematical maturity, but more especially one of time schedules. In defense of the earlier introduction of a few logical proofs the argument has been advanced that numerous pupils who leave school at the end of the ninth year should not be deprived of a bowing acquaintance with that unique spirit which has been the glory of demonstrative geometry. A very beautiful conception; but is such a procedure really feasible under existing conditions? Is it pedagogically worth while? The opposing argument has been offered that for similar reasons one might desire to give to these departing boys and girls a few lessons in Greek, or physics, or harmony, or a rapid panoramic view of the world's other great intellectual treasures. After all, there will always be those who turn from the arduous cultivation of the mind to fields of immediate utility.

Conclusion.

The brief review attempted above intends to convey the impression that, on the whole, the tentative syllabus submitted by the National Committee is a sane and constructive piece of work. It is in harmony with the experience and the ideals of progressive teachers, and it can therefore be regarded as a reliable guide to those who wish to assist in the future development of junior high school mathematics.

ROCHESTER, N. Y. WILLIAM BETZ.

DISCUSSION OF PART VI.

A discussion of the Preliminary Report of the National Committee seems to the writer to be necessarily commendatory in its nature. Comment seems like painting the lily white. The report is a great forward step in our profession.

Part VI, to which I would call attention for the moment, gives additional evidence of the good sense of the document in suggesting several arrangements of the material. This makes the course adaptable to a great variety of schools and conditions.

Personally, I covet the privilege of recommending most heartily Plan A. Its appeal is logical and pedagogical. It is logical intrinsically and in its relation to the purposes of the Junior High School. It is pedagogical in its psychological arrangement of the material.

The child in the seventh school year is mentally able to handle such arithmetic as is needed in practical applications to home and school life. He, furthermore, is beginning to have a care for such problems. He can grapple with simple budget making and percentage relationships and should be encouraged to do so.

Also at this stage in his career he needs training in handwork and the arrangement of his written material. He is interested in form—size and shape. A fortuitous combination of training in these directions finds its expression in Intensive Geometry.

When the pupil has reached the eighth year, he is ready to generalize his views to a certain extent. He can look at abstractions as such to a limited degree. Therefore, he can enlarge the scope of his seventh year problems and classify his facts. He is ready to make and use formulas and equations. He is ready to get the concept of function. He is ready to express relationship graphically. In short, he is mentally capable of and is interested in algebra as a shorthand and a problem solver.

In his ninth school year, the student has acquired a beginning of a logical sense and is entitled to a peep into demonstrational geometry. Obviously, this subject for pupils of this age must be approached very gradually. Moreover, it must have a practical aspect by means of the type of exercises selected. It must also be given with a very limited syllabus. In short, an extensive view of the subject should be given in this year and should not be the final treatment of the work in the secondary school. The report calls for trigonometry. This undoubtedly implies merely a chapter of Plane Geometry applying the work of similar triangles to the solution of simple trigonometric problems.

Thus, in steps adapted to the interest and maturity of the pupil, the types of secondary school mathematics have been prescribed to the student of the Junior School. Opportunity has been given to find his interest and his ability. The mathematics of the secondary school has been presented in extensive fashion preliminary to the intensive study of these same topics in the Senior High School.

Should the school career of the pupil be terminated at the conclusion of his Junior High School, he has been put in possession of the mathematical tools for practical work. Herein, another purpose of the Junior High School has been fulfilled.

It is interesting to note that in achieving as hinted the purposes of the Junior High School, the subject of mathematics has not been logically maltreated. The sequence of topics, for example, is quite as logical as pedagogical and psychological.

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DISCUSSION OF THE FOUR PRECEDING ARTICLES.

The publications of the National Committee on Mathematical Requirements represent either detailed investigations, or else the composite thought of a very large number of teachers who cooperated with the Committee. The discussion by a member of the Committee can express only the opinion of that individual. With these important facts in mind, it may be worth while to review the preceding articles, and to summarize the points which imply some degree of difference of opinion.

Turning to the first of these articles by Mr. Breslich, we come sharply to a vital issue,—correlation. Mr. Breslich says: "Although correlation is not advocated by the Committee, the second, third and fourth points indicate the possibility, or even the advisability of correlating algebra, geometry, arithmetic and trigonometry," and later, "the report does not show that the Committee have considered the plan, at least not sufficiently to include it as one of the possible arrangements worked out in part six of the report."

This conclusion is clearly in error. "Plan E" of Part VI consists of the following:

> "Plan E."
>
> "First year: Intuitive geometry, simple formulas, elementary principles of statistics; arithmetic (as in Plan A).
>
> Second year: Intuitive geometry; algebra.
>
> Third year: Intuitive geometry; numerical trigonometry; arithmetic."
>
> "By this plan the work of the first two and one half years may be described as general mathematics, while the last half year would be devoted to the special civic and economic features of business practice."

Surely this plan at least demands correlation as one solution. Incidentally, it is to be hoped that the minutes of the National Committee will show that "Plan E" as printed involves an error in proofreading. It would seem that in addition to the contents stated, the second year should include arithmetic also, and that algebra should be added to the third-year list.

Even if "Plan E" were entirely omitted, Mr. Breslich's concern would not be justified. In all the plans proposed, the work of each year is not to be devoted wholly to a single field such as arithmetic or algebra. Keeping in mind that there is no attempt to prescribe the order of presentation of the topics included, there is certainly ample freedom for the ideas and principles of the various fields to be so taught as to reinforce each other.

The real vital issue so far as concerns Mr. Breslich is the fusion of demonstrative geometry with materials chosen from arithmetic, algebra and the like. If the intuitive geometry of the report is taught, the pupil will come to the consideration of demonstrative geometry knowing most of the vocabulary and the symbolism. Moreover, he will have gained conviction of many of the significant geometric relations, and will also have acquired some facility in their use.

Most teachers will probably feel that the only thing worth while (*if any*) that is left is the logical unity, and the method of attack in solving problems dealing with more or less similar materials and situations. Is anything gained in this development by constant interruptions of materials chosen from algebra or other fields? This is the debatable issue on which the National Committee takes no clear definite position. The matter must be settled by scientific experiment, rather than by the assertion of one individual, or that of a group. Possibly the most helpful part of the report on this question is the sentence on page 9: "It is necessary, however, that those propositions which are to be used as the basis of subsequent formal proofs be explicitly listed and their logical significance recognized."

Miss Gugle issues a word of warning "not to over stress such topics as indirect cases of percentage, foreign money, stocks and bonds, and statistics." Since no one possesses reliable information concerning the social values of the topics in arithmetic, one "warning" is as good as another. "Statistics" seem to show that at present (as well as for a generation past) a considerable portion of our citizens have stock and bond investments. In the discussion of the cases of factoring, Miss Gugle seems to express no real disagreement concerning the content. Her statements appear to deal with the method of teaching these cases, a matter which the Committee did not include.

On the unit of demonstrative geometry, Miss Gugle says, among other things, that "care should be taken to choose theorems in which the pupil has something to prove, not self-evident facts and assumptions." It is precisely to make this possible that we read on page 9 of the report: "It should be noted that from the strictly logical point of view the attempt to reduce to a minimum the list of axioms, postulates, or assumptions is not at all necessary, and from a pedagogical point of view such an attempt is very undesirable."

Mr. Betz questions "the obligatory use of the T-square and triangle." Possibly, he is correct, but it would seem that most communities could furnish a T-square, and that usually some boy would be willing to make a simple wooden or cardboard triangle.

We may leave this, and all other matters of minor importance in Mr. Betz's paper, and come to the consideration of his paragraph on demonstrative geometry. His first sentence, "there are a few good American schools, usually private or endowed, that succeed in doing some excellent work in demonstrative geometry in the 9th school year," may be questioned on the basis of fact. It is sufficient to point out that the junior high school series of mathematics texts, which enjoys widest use in *public* schools to-day, contains a unit of demonstrative geometry. Later in the paragraph we read "the opposing argument has been offered that for similar reasons one might desire to give to these departing boys and girls a few lessons in Greek or physics or harmony, or a rapid panoramic view of the world's other great intellectual treasures." While many mathematics teachers (including the writer) are not fully convinced of the wisdom of including demonstrative geometry in the junior high school, the issue certainly demands less fallacious argument. "A few lessons in Greek, or physics or harmony" are wholly false analogies, whereas, if by "a rapid panoramic view of the world's great intellectual treasures" is meant a preliminary consideration of the simple and significant principles of the various fields of human achievement, as, for example, science, art, and the like, the project may be both feasible and desirable. Mr. Wells' recent attempt to give the general reader this panoramic view in "The Outline of History" suggests a task that seems worth doing in all fields, including history, if it should turn out that Mr. Wells has not arrived.

Mr. Walsh, in recommending "Plan A" sees in the work of the National Committee an achievement which it did not consciously attempt. He says, "Its (Plan A) appeal is logical and pedagogical," and in the last paragraph of his paper we read, "the sequence of topics (referring to junior high school mathematics) is quite as logical as pedagogical, and psychological." If all this is true, it must be so by chance, for the Committee did not consciously attempt to secure these high values. It realized that much detailed and exacting work must be done in the class rooms with children by many groups of teachers before the program, suggested as the outcome of composite opinion, can be realized. It is necessary to emphasize here that the Committee did not attempt to list the order of topics, or even the order in which arithmetic, algebra and geometry should be pursued in any one year of "Plan A," or in any other plan.

It is gratifying to see to what extent differences concerning the junior high school problem are being ironed out. This happy outcome is largely due to such thoughtful attention and constructive critcism as are illustrated by the four preceding papers. The outlook for secondary school mathematics is very promising, and of the various divisions, that of the junior high school is particularly attractive.

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